MA 591-003 Special Topics: Fall 2017 CONVEX OPTIMIZATION (Semidefinite Programming)

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Overview: Convex optimization is a rich and thriving area of mathematics, and convex optimization methods have become an essential part of the toolkit of computational math and engineering, with applications ranging from data science to medicine and health care, finance, operations research, theoretical computer science, and virtually every branch of engineering. This course will develop the mathematical and algorithmic fundamentals of the area, illustrated with applications and open research problems fitting the audience's background.

Our primary objects of interest will be semidefinite and second-order cone programming problems. These are the largest classes of optimization problems for which we have the tools to compute high-accuracy solutions of very large scale problems quickly. Virtually every successful application of convex optimization is an instance of semidefinite programming.

Prerequisites: The course will be self-contained, and start from the beginning, assuming only undergraduate linear algebra. In particular, previous exposure to optimization or numerical computing will be helpful, but will not be necessary. The applications discussed will be kept basic, and tailored to the background of the audience.

Reference material: There is no official textbook, but several texts are available that can serve as auxiliary reference books. (You won't need to consult them unless you want to.)

- Ben-Tal and Nemirovski: Lectures on Modern Convex Optimization
- Boyd and Vandenberghe: Convex Optimization
- Blekherman, Thomas, and Parrilo: Semidefinite Optimization and Convex Algebraic Geometry
- Borwein and Lewis: Convex Analysis and Nonlinear Optimization
- Ruszczynski: Nonlinear Optimization

Course objectives: Upon completing the course, you will

- recognize convex optimization and semidefinite programming problems that arise in your field,
- understand the basic mathematical theory of such problems, concentrating on results that translate to practical computation,
- know how to solve these problems,
- have the background to use the methods and underlying ideas in your own research,
- have the background to start your own research in convex optimization.

Topics covered in the class include:

- The geometry of convex sets and convex cones. The fundamentals of convex analysis.
- Optimization problems over convex cones. Applications from a variety of fields.
- Second-order cone representable functions and sets.
- Semidefinite representable functions and sets.
- Duality; theorems of the alternative. The Conic Farkas Lemma and its applications.
- Geometry of convex optimization: convex separation and the ellipsoid method, and their impact on the computational complexity of optimization.
- Second-order (interior-point) methods for semidefinite and second-order cone programming.
- First-order methods for semidefinite and second-order cone programming.
- Combinatorial optimization: semidefinite relaxations and approximation algorithms.
- Symmetric cones and Euclidean Jordan algebras.
- Polynomial optimization and sums-of-squares. Applications.

#	DATE	TOPIC
1	Wednesday, August 16, 2017	Introduction. Convex and conic optimization within the broader context of optimization models
2	Friday, August 18, 2017	Introduction to convexity and to positive semidefinite matrices
3	Monday, August 21, 2017	
4	Wednesday, August 23, 2017	
5	Friday, August 25, 2017	Farkas Lemma and duality.
6	Monday, August 28, 2017	
7	Wednesday, August 30, 2017	
8	Friday, September 1, 2017	
	Monday, September 4, 2017	Nonnegative polynomials, moment vectors, and their semidefinite representation.
9	Wednesday, September 6, 2017	Labor day
10	Friday, September 8, 2017	The second-order cone. Second-order cone representable sets and functions. Applications.
11	Monday, September 11, 2017	
12	Wednesday, September 13, 2017	
13	Friday, September 15, 2017	Semidefinite representable sets and functions. The Schur complement lemma. Applications.
14	Monday, September 18, 2017	
15	Wednesday, September 20, 2017	
16	Friday, September 22, 2017	Polynomial time algorithms; introduction. Computational models and their importance in numerical algorithms.
17	Monday, September 25, 2017	The max cut problem.
18	Wednesday, September 27, 2017	
19	Friday, September 29, 2017	The separation problem for convex sets and the ellipsoid method
20	Monday, October 2, 2017	
21	Wednesday, October 4, 2017	
	Friday, October 6, 2017	Fall break
22	Monday, October 9, 2017	Alternating projections and the ADMM.
23	Wednesday, October 11, 2017	
24	Friday, October 13, 2017	First-order methods for convex/conic optimization.
25	Monday, October 16, 2017	
26	Wednesday, October 18, 2017	
27	Friday, October 20, 2017	Interior-point methods and software (for LP, SOCP, SDP, general convex/conic programming)
	Monday, October 23, 2017	no class
28	Wednesday, October 25, 2017	
29	Friday, October 27, 2017	
30	Monday, October 30, 2017	
31	Wednesday, November 1, 2017	
32	Friday, November 3, 2017	
33	Monday, November 6, 2017	Sum-of-squares optimization, integer programming, and semidefinite hierarchies
34	Wednesday, November 8, 2017	

35 Friday, November 10, 2017
36 Monday, November 13, 2017
37 Wednesday, November 15, 2017
38 Friday, November 17, 2017 Final presentations
39 Monday, November 20, 2017
Wednesday, November 22, 2017 Thanksgiving
Friday, November 24, 2017 Thanksgiving
40 Monday, November 27, 2017
41 Wednesday, November 29, 2017
42 Friday, December 1, 2017